# Coincidence and Common Fixed Points of Infinite Family of Mappings Under Weaker Conditions

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**Abstract**

In this paper, some common fixed point theorems for pairs of subcompatible and reciprocally continuous mappings or compatible and subsequentially continuous mappings satisfying integral type are obtained. Our results improve several results especially the results of [2], and others.

**Key words and phrases**: Compatible (resp. subcompatible, reciprocally continuous, subsequentially continuous) mappings, metric space.

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## 1 Introduction

We want list only the following needed definitions.

**Definition 1** *([3]) Two self-mappings $f$ and $g$ of a metric space $(\mathcal{X}, d)$ are called* ***compatible*** *if and only if*

$$\lim_{n\to\infty} d(fgx_n, gfx_n) = 0,$$

*whenever $\{x_n\}$ is a sequence in $\mathcal{X}$ such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in \mathcal{X}$.*

**Definition 2** *([1, 2]) Let $f$ and $g$ be two self-mappings of a metric space $(\mathcal{X}, d)$. $f$ and $g$ are* ***subcompatible*** *if and only if there exists a sequence $\{x_n\}_{n\in\mathbb{N}}$ in $\mathcal{X}$ such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in \mathcal{X}$ and $\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$.*

**Definition 3** *([4]) Two self-mappings $f$ and $g$ of a metric space $(\mathcal{X}, d)$ are called* ***reciprocally continuous*** *if $\lim_{n\to\infty} fgx_n = ft$ and $\lim_{n\to\infty} gfx_n = gt$ whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t$ in $\mathcal{X}$.*

**Definition 4** *([1, 2]) Two self-mappings $f$ and $g$ of a metric space $(\mathcal{X}, d)$ are said to be* ***subsequentially continuous*** *if and only if there exists a sequence $\{x_n\}$ in $\mathcal{X}$ such that $\lim\limits_{n\to\infty} fx_n = \lim\limits_{n\to\infty} gx_n = t$ for some $t$ in $\mathcal{X}$ and satisfy $\lim\limits_{n\to\infty} fgx_n = ft$ and $\lim\limits_{n\to\infty} gfx_n = gt$.*

Now, we state our main results.

## 2 A general common fixed point theorem

**Theorem 5** *Let $f$, $g$, $h$ and $k$ be four mappings of a metric space $(\mathcal{X}, d)$ into itself such that the pairs $(f, h)$ and $(g, k)$ are subcompatible and reciprocally continuous. Let $\psi : \mathbb{R}_+ \to \mathbb{R}$ be a Lebesgue-integrable mapping which is summable and let $\varphi : \mathbb{R}_+^6 \to \mathbb{R}_+$ be a real continuous function satisfying the following condition:*

$$(\varphi_1) : \int_0^{\varphi(u,u,0,0,u,u)} \psi(t)dt > 0 \text{ for } u > 0.$$

*If, for all $x$ and $y \in \mathcal{X}$,*

$$\int_0^{\substack{\varphi(d(fx,gy),d(hx,ky),d(fx,hx),\\ d(gy,ky),d(ky,fx),d(hx,gy))}} \psi(t)dt \leq 0, \tag{2.1}$$

*then, $f$, $g$, $h$ and $k$ have a unique common fixed point.*

Now, we give a generalization of the above theorem.

**Theorem 6** *Let $h$, $k$ and $\{f_n\}_{n=1,2,\ldots}$ be mappings from a metric space $(\mathcal{X}, d)$ into itself such that*

*$(i)$ the pairs $(f_n, h)$ and $(f_{n+1}, k)$ are subcompatible and reciprocally continuous,*

*$(ii)$ the inequality*

$$\int_0^{\substack{\varphi(d(f_nx,f_{n+1}y),d(hx,ky),d(f_nx,hx),\\ d(f_{n+1}y,ky),d(ky,f_nx),d(hx,f_{n+1}y)))}} \psi(t)dt \leq 0$$

*holds for all $x$, $y$ in $\mathcal{X}$, $\forall n = 1, 2, \ldots$, where $\varphi$ and $\psi$ are as in theorem 5, then, $h$, $k$ and $\{f_n\}_{n=1,2,\ldots}$ have a unique common fixed point.*

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