Optimal Control of Dynamic Bipartite Matching Models

Arnaud Cadas, Josu Doncel and Ana Bušić

January 31, 2020

Optimal Control of Dynamic Bipartite Matching Models

Arnaud Cadas[12] Josu Doncel[3] Ana Bušić[12]

[1] INRIA, Paris, France.
arnaud.cadas@inria.fr
ana.busic@inria.fr
[2] DI ENS, CNRS, PSL Research University, Paris, France.
[3] University of the Basque Country, UPV-EHU, Leioa, Spain.
josu.doncel@ehu.eus

Mots-clés : *Dynamic matching models, Markov decision processes, Optimal control*

1 Introduction

The theory of matching started with Peterson and König and was under a lot of interests in graph theory with problems like maximum matchings. It was extended to online matching setting [6] where one population is static and the other arrives according to a stochastic process. In the recent years, fully dynamic matching models have been considered where both populations are random. The importance of matching models was shown through applications in various fields such as health [1] or ridesharing [2].

We study matching models from a queueing theory perspective, where a supply item and a demand item arrive to the system at each time step and can be matched according to a compatibility graph or stay in buffers. [7, Theorem 1] proves that in a matching model where items arrive one by one, there exists no arrival distribution which verifies the necessary stability conditions for bipartite matching graphs. This result justifies why we assume arrivals by pairs as in [4, 3]. We consider that there is a holding cost that is a linear function of the buffer sizes. Our objective is to find the optimal matching policy in the discounted cost problem and in the average cost problem for general bipartite matching graphs. For this purpose, we model this problem as a Markov Decision Process.

Optimality results are scarce, and have been derived for some matching models in the asymptotic regimes. An extension of the greedy primal-dual algorithm was developed in [8] and was proved to be asymptotically optimal for the long-term average matching reward. However, they considered rewards on the edges, which differs from our model with holding costs. In [4], the authors considered the asymptotic heavy-traffic setting, and identified a policy that is approximately optimal with bounded regret using a workload relaxation approach. We consider the non-asymptotic setting, i.e. we allow for any arrival rates under the stability conditions established in [3].

2 Contributions

We first consider a matching model with two supply and two demand classes that has a N-shaped matching graph. For this system, we show that the optimal matching policy is of threshold type for the diagonal edge and with priority to the end edges of the matching graph. To prove this result, we use the method of structured policies as described in [9, Section 6.11]. We also compute the optimal threshold for a linear cost function in the case of the average cost problem. This part of the work has already been published in [5].

Using our insight from the N-shaped matching graph, we study more general bipartite matching graphs under some assumptions on the costs.

For complete graphs minus one edge, their structure is very similar to the N-shaped graph. We can define a projection from the former to the latter by considering the two nodes tied to the missing edge as the end nodes of the N-shaped graph and aggregating the others. Using this projection and our results on the N-shaped graph, we prove that any policy which is of threshold type for the diagonal edge and with priority to the end edges after being projected is optimal. This result assumes that for any state of the system, its holding cost is equal to the holding cost of its projection.

For arbitrary acyclic graphs, we define extreme edges which are edges such as one of the two nodes has a degree one. We prove that it is optimal to prioritize and match everything we can in those edges under the assumption that there is a higher holding cost on those edges compared to their neighbors (all the edges with at least one node in common). This result uses a similar technique as the one used on the N-shaped graph.

Finally, we present our work in progress for the W-shaped matching graph where we conjecture that a threshold type policy with priority in the extreme edges is optimal under the assumption of higher cost on the extreme edges. We define the properties needed to prove the optimality of our conjecture. We prove that under those properties our conjecture is the optimal decision rule. We discuss the challenges that arise to prove the preservation of those properties by the dynamic programming operator. We also present numerical experiments which suggest that matching in priority the extreme edges is not optimal when we remove the assumption of higher cost on those edges.

Références

[1] Itai Ashlagi, Patrick Jaillet, and Vahideh H. Manshadi. Kidney exchange in dynamic sparse heterogenous pools. In *Proceedings of the Fourteenth ACM Conference on Electronic Commerce*, EC '13, pages 25–26, New York, NY, USA, 2013. ACM.

[2] Siddhartha Banerjee, Yash Kanoria, and Pengyu Qian. The Value of State Dependent Control in Ridesharing Systems. *ArXiv e-prints*, March 2018.

[3] Ana Busic, Varun Gupta, and Jean Mairesse. Stability of the bipartite matching model. *Advances in Applied Probability*, 45(2) :351–378, 2013.

[4] Ana Busic and Sean P. Meyn. Approximate optimality with bounded regret in dynamic matching models. *ArXiv e-prints*, 2014.

[5] Arnaud Cadas, Ana Bušić, and Josu Doncel. Optimal control of dynamic bipartite matching models. In *Proceedings of the 12th EAI International Conference on Performance Evaluation Methodologies and Tools*, VALUETOOLS 2019, pages 39–46, New York, NY, USA, 2019. ACM.

[6] Jon Feldman, Aranyak Mehta, Vahab S. Mirrokni, and S. Muthukrishnan. Online stochastic matching : Beating 1-1/e. In IEEE Computer Society, editor, *Proceedings of the 2009 50th Annual IEEE Symposium on Foundations of Computer Science*, pages 117–126. ACM Press, 2009.

[7] Jean Mairesse and Pascal Moyal. Stability of the stochastic matching model. *Journal of Applied Probability*, 53(4) :1064–1077, 2016.

[8] Mohammadreza Nazari and Alexander L. Stolyar. Reward maximization in general dynamic matching systems. *Queueing Systems*, Nov 2018.

[9] Martin L. Puterman. *Markov decision processes : discrete stochastic dynamic programming.* Wiley series in probability and statistics. Wiley-Interscience, 2005.