De-noising of Vibration Signals of Induction Motor using Compressed Sensing.

Niraj Kumar

December 2, 2019

De-noising of Vibration Signal of Induction Motor using Compressed Sensing

Niraj Kumar

Department of Electronics & Communication Engineering, Sagar Institute of Research & Technology Bhopal (M.P.), India

ABSTRACT: - This paper proposes a de-noising method for vibration signal based on compressed sensing. Compressed sensing technology is a new method for de-noising a vibration signal, in the field of signal processing. It can achieve data acquisition as well as data compression at the same time. In this paper first, the original signal express into a low dimensional space. According to the compressed sensing theory the uncontaminated vibration signal can be represented sparsely by some transform domain while noise can't be represent, therefore the noise information in original signal can be banished by compressed sensing. Then the original signal can be recovered by reconstruction algorithm. The orthogonal matching pursuit algorithm used in this paper and finally de-noising is achieved. This method is verified by induction motor vibration signal which polluted by white Gaussian noise. The simulation results show that the SNR could be improved and signal reconstruction error is minimum when we set appropriate value of sparsity and linear measurement in OMP algorithm.

Keywords: - Vibration signal; Compressed sensing; De-noising; Sparse signal; Dictionary matrix; Gaussian random matrix; OMP algorithm

1. INTRODUCTION

The study of condition monitoring of machine and fault diagnosis is a challenging task. It is always necessary to collect the vibration signal or data to achieve the vibration analysis. Vibration analysis is used to condition monitoring of machine. Bearing faults in machine appear transient periodically in the vibration data. There are many types of faults can appear in the machine. The bearing is an essential component of the machine. The bearing failure mostly occurs due to temperature, lubricant problem, corrosion etc [10]. The bearing faults in machine increased, so this is not good for the health of the machine. For this reason detection, de-noising and analysis of such type of irregular vibration signal are necessary. In the process of data, acquisition noise might be brought in inevitably for the influence of the environment, temperature and sensors installation state and so on. Due to this interference information in fault detection or analysis of vibration signal. Therefore, it is essentially important to carry out de-noising the vibration signal before its analysis. There are many types of general de-noising methods for vibration signal are mathematics morphology filtering, singular value decomposition, empirical mode decomposition and wavelet de-noising and so on. In the wavelet de-noising, there is a problem of threshold setting, it can affect the noise reduction performance significantly, which should be a great problem solved.

Compressed sensing (CS) is a new sampling theory which can overthrow the traditional Nyquist Shannon sampling theory and was proposed by Donoho and Candes et al in 2006 [1,2]. The process of CS may be divided into two steps. First combining the sampling with compressing, we can obtain the non-adaptive linear projection (or measurement) of the original signal. Then we can use suitable recovery algorithm for recovering the original signal with the help of these measurements [4, 9]. The application of compressive sensing on image de-noising [5] and speech enhancement [7] etc. Had been studied since the compressed sensing theory appears. These previous work help us to present a de-noising method for vibration signal based on compressed sensing theory. The compressed sensing de-noising method is similar with the wavelet de-noising, while it can avoid the threshold setting and achieve de-noising by compressed sensing and reconstruct the vibration signal. The work is divided into five sections in this paper. Section second describes compressed sensing and sparse representation; Section third describes compressed sensing de-noising method and experimental tests described in section four. Work is concluded in section Five.

2. COMPESSED SENSING AND SPARSE REPRESENTATION

Sparse signal: - As we know that most nature signal in time domain are not absolutely sparse, but they can be represent approximate sparely by some transform domains such as Fourier domain, Wavelet domain, Qubo domain and DCT domain. Now consider the signal $\boldsymbol{S} \in \boldsymbol{R}^P$ and the vector $\{\psi_i\}_{i=1}^{P}$ as a column vector to form $P * P$ basis matrix $\boldsymbol{\Psi} = [\boldsymbol{\psi_1}, \boldsymbol{\psi_2} \ldots\ldots \boldsymbol{\psi_P}]$, Then any signal $\boldsymbol{S}$ can be expressed as

$$\boldsymbol{S} = \sum_{i=1}^{P} \boldsymbol{x}_i\, \boldsymbol{\psi}_i \qquad (1)$$

Where the coefficient $\boldsymbol{x_i} = \langle \boldsymbol{S},\ \boldsymbol{\psi_i} \rangle = \boldsymbol{\psi_i'S}$ and Eq. (1) can be represented in the matrix form as

$$\boldsymbol{S} = \boldsymbol{\Psi X} \qquad (2)$$

Where $\boldsymbol{X} = [\,\boldsymbol{x_i}] = [\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots .\, \boldsymbol{x_p}]$ is the projection coefficient or sparse signal and obviously $\boldsymbol{S}$ and $\boldsymbol{X}$ is the equivalent representation and $\boldsymbol{X}$ is the $\boldsymbol{\Psi}$ domain representation. The $\boldsymbol{\Psi}$ is taken as a dictionary matrix (discrete cosines transform, DCT matrix). The signal $\boldsymbol{S}$ is called compressible signal if $\boldsymbol{X}$ has only few non-zero element, also if $\boldsymbol{X}$ is called $\boldsymbol{k}$ sparse signal if $\boldsymbol{X}$ has only k non-zero coefficient.

Compressed sensing principle: - Consider a signal $\boldsymbol{S} \in \boldsymbol{R}^P$, First we want to obtain the linear projection of original high dimension signal $\boldsymbol{S}$ into a low dimension signal $\boldsymbol{Y}$, by using a measurement matrix $\boldsymbol{\phi} \in \boldsymbol{R}^{p*p}$(M<<P). And each row of matrix $\boldsymbol{\phi}$ can be taken as a sensor which multiply with the signal $\boldsymbol{S}$ and obtain the part of the signal information. Now we get compressive measurement of signal $\boldsymbol{S}$ as shows in bellow

$$\boldsymbol{Y} = \boldsymbol{\phi}\, \boldsymbol{S} \qquad (3)$$

Now we get the linear measurement $Y \in R^M$. But if we want to recover the signal $\boldsymbol{S}$ from $\boldsymbol{Y}$. These fewer linear measurement should contain the enough information to recover the signal $\boldsymbol{S}$ from $\boldsymbol{Y}$. According to linear algebra the Eq. (3) has so many solutions, there for we can't reconstruct the original signal $\boldsymbol{S}$ from $\boldsymbol{Y}$ when (M<<P) however, if $\boldsymbol{S}$ is sparse some transform domain $\boldsymbol{\Psi}$, then number of unknown will reduce and it will be possible to reconstruct the signal $\boldsymbol{S}$ from $\boldsymbol{Y}$. Now combine the Eq. (2) and (3) then we will get

$$\boldsymbol{Y} = \boldsymbol{\phi \Psi X} \qquad (4)$$

Where $\boldsymbol{X}$ is sparse signal and denoting $\boldsymbol{B} = \boldsymbol{\phi\Psi}$ then Eq. (4) can be written as

$$\boldsymbol{Y} = \boldsymbol{BX} \qquad (5)$$

Now it is possible to recover the signal $\boldsymbol{S}$ from $\boldsymbol{Y}$. Model structure of compressed sensing shows in Fig. (1).

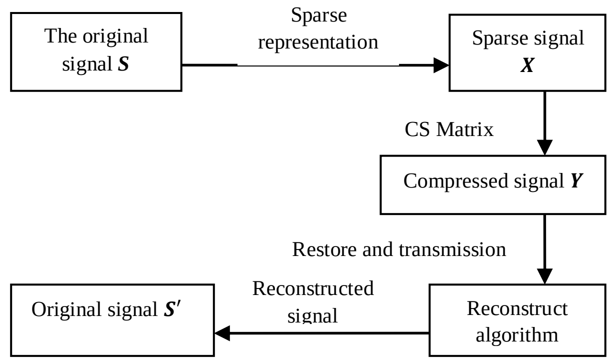

Fig.1. Model structure of compressed sensing

Selection of measurement matrix: - For reconstructing the original signal. Candés and Tao presented also proved that the CS matrix $\boldsymbol{B}$, must be satisfy the limited isometric nature (Restricted Isometry Property) [9,13] that means for any $\boldsymbol{k}$ sparse signal $\boldsymbol{X}$, should meet Eq. (6)

$$(\boldsymbol{1} - \boldsymbol{\delta_k}) \leqslant \frac{\|\boldsymbol{BX}\|_2^2}{\|\boldsymbol{X}\|_2^2} \leqslant ((\boldsymbol{1} + \boldsymbol{\delta_k}) \qquad (6)$$

Where $\boldsymbol{\delta_k} \in (0,1)$, to meet the RIP condition is comparatively difficult. There for Baraniuk proposed the irrelevance between observation matrix $\boldsymbol{\phi}$ and the dictionary matrix $\boldsymbol{\Psi}$ all most equal to the RIP [11], the relation between $\boldsymbol{\phi}$ and $\boldsymbol{\Psi}$ shows in Eq. (7)[15]

$$\mu(\boldsymbol{\phi}, \boldsymbol{\Psi}) = \sqrt{P} \cdot \max|\ \boldsymbol{\phi}_i^T\, \boldsymbol{\Psi}_j|\ 1 \le i \le M, 1 \le j \le P \ (7)$$

Where $\boldsymbol{\phi}_i^T$ is the i^{th} row of $\boldsymbol{\phi}$ and $\boldsymbol{\Psi_j}$ is the $\boldsymbol{j^{th}}$ column of $\boldsymbol{\Psi}$. If these conditions are satisfy then the original signal can be definitely recovered according to Eq. (2) and Eq. (4).The matrix form of compressive sensing shown in bellow.

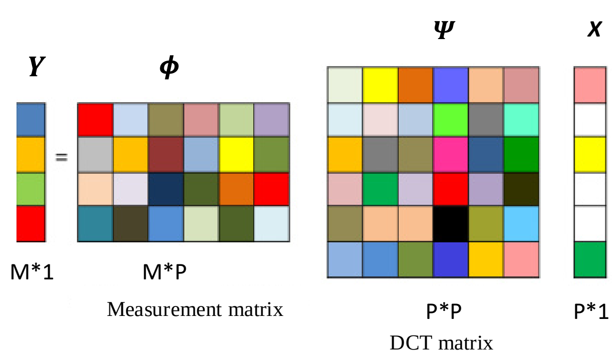

Fig.2. Matrix form of compressive measurement

Now, we select the orthogonal matching pursuit (OMP) algorithm and thus achieve the reconstruction of the original signal [3, 12].

Algorithm:-OMP algorithm for the signal reconstruction.

INPUT: -Observation matrix $\boldsymbol{Y}$, CS matrix $\boldsymbol{B}$, and sparsity $\boldsymbol{K}$

OUTPUT: -Estimate value of signal S^j, residual errorR^j

INITIALIZATION: - $\boldsymbol{S}^{[0]} = 0$,$\boldsymbol{R}^{[0]} = \boldsymbol{Y}$, $\boldsymbol{A}_0^{[0]} = \emptyset$

For j=1, j=j+1; as soon as the criteria is satisfy

- ❖ Find the residual error $\boldsymbol{R}^{[j-1]}$ and the inner product $\boldsymbol{a}^j = \boldsymbol{B}^T\boldsymbol{R}^{[i-1]}$;
- ❖ Allocate the element of a^j

  $\boldsymbol{L}^{[j]} = argmax| \boldsymbol{a}_L^{[j]}| /\|\boldsymbol{B}_L\|_2$;
- ❖ Expansion of the index $\boldsymbol{A}_0^{[j]} = \boldsymbol{A}_0^{[j-1]} \cup \boldsymbol{L}^{[j]}$;
- ❖ $\boldsymbol{S}^{[j]}$=$\boldsymbol{B}^{\dagger}\boldsymbol{A}_0^{[j]}$ $\boldsymbol{Y}$, and "†"denote pseudo-inversing;
- ❖ Upgrade the residual error $R^{[j]} = Y - BS^{[j]}$;

End For

---

3. COMPRESSED SENSING DE-NOISING METHOD

A de-noising method for vibration signal based on compressed sensing will be analysis in this portion. The vibration signals were acquired from the induction motor (single phase) under loaded condition. The induction motor was run with healthy bearing (6203-2Z-H320B-JEM-SKF) was shown in Fig.3, the vibration signal of healthy bearing was taken by accelerometer (ESPL3X15) [6]. The simulation time was set as 2 second. The vibration time signal of healthy bearing shown in Fig.5. The faulty bearing and there vibration time signal was shown in Fig.4 and Fig.6 respectively.

Fig.3. Healthy bearing

Fig.4. Faulty bearing

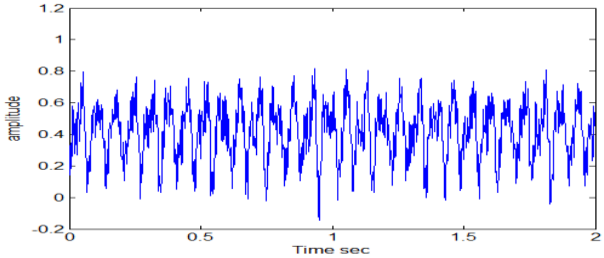

Fig.5. Healthy vibration time signal

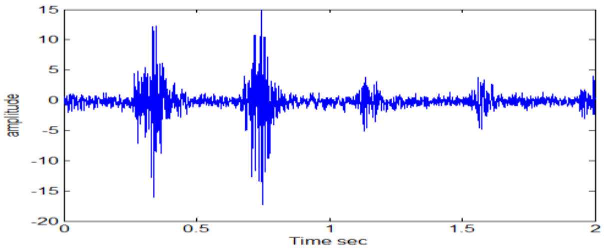

Fig. 6. Faulty vibration time signal

Generally, the nature signals are not absolutely sparse. But they can be represented sparsely by some transform domain $\boldsymbol{\Psi}$ such as discrete cosine transforms (DCT). We select a signal $\boldsymbol{S} \in \boldsymbol{R}^p$ (P=1200) from the signal shown in Fig. 7, shows uncontaminated vibration signal and The DCT result of signal S is shown in Fig.8. We can see from fig.8, that the almost coefficient after applying DCT are close to zero, that means the signal $\boldsymbol{x}$ can be represented nearly sparse. Signal sprsity is the essential condition for compressed sensing de-noising. In general the signal $\boldsymbol{S} \in \boldsymbol{R}^p$ can be compressible (sparsely) in domain $\boldsymbol{\Psi}$.

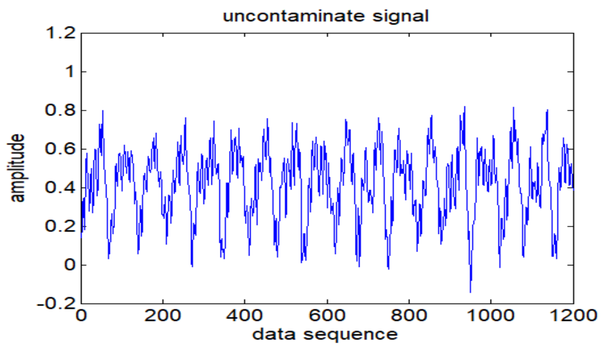

Fig.7. Uncontaminated vibration signal

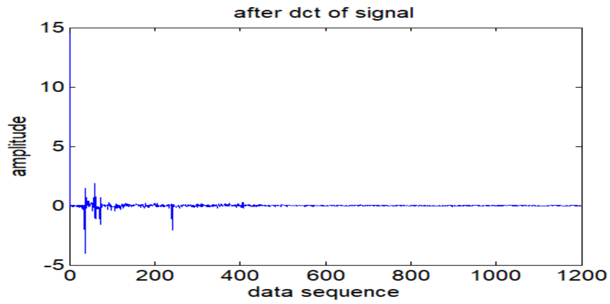

Fig.8. Corresponding coefficient after DCT

Now create the white Gaussian noise $\boldsymbol{w} \in \boldsymbol{R}^p$. According to the compressed sensing theory noise can't be compressible in domain $\boldsymbol{\Psi}$. If the signal $\boldsymbol{S}$ is polluted by white Gaussian noise $\boldsymbol{w}$. Then we can obtained $(\boldsymbol{S}+\boldsymbol{w}) \in \boldsymbol{R}^p$. Consider the measurement matrix $\boldsymbol{\phi} \in \boldsymbol{R}^{M*P}$(M<P), then the contaminate vibration signal projected high dimension space to low dimension space and obtained linear observation $Y = \boldsymbol{\phi}(\boldsymbol{S}+\boldsymbol{w})$ and sparse form of signal $\boldsymbol{S}$ and noise $\boldsymbol{w}$ are X_s and X_w respectively. Then observation $\boldsymbol{Y}$ can be written as

$$\boldsymbol{Y} = \boldsymbol{B}(\boldsymbol{X_s}+\boldsymbol{X_w}) \tag{8}$$

Where $\boldsymbol{B} = \boldsymbol{\phi\Psi},$ implemented model of compressed sensing de-noising shown in Fig.9.

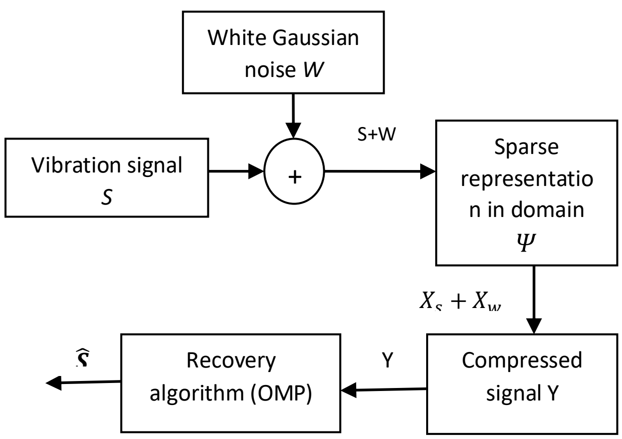

Fig.9. Implemented model of compressed sensing de-noising

4. EXPERIMENTAL RESULTS

The vibration signal shown in Fig.7. Will be used as a uncontaminated signal $\boldsymbol{S} \in \boldsymbol{R}^P$ (P=1200), and the white Gaussian noise will be used to polluted the vibration signal, then we obtained contaminated signal $\boldsymbol{sw} \in \boldsymbol{R}^p$. The $\boldsymbol{\Psi}$(DCT matrix) and $\boldsymbol{\phi}$(Gaussian random matrix**)** will be taken as a dictionary matrix and the measurement matrix respectively. The OMP algorithm will be used for signal reconstruction. To evaluate the simulation results of de-noised vibration signal $\boldsymbol{S}$. The compression ratio of the signal [16] and the signal to noise ratio (SNR) of the contaminated vibration signal and de-noise signal are calculated respectively [3].

$$\boldsymbol{CS} = \frac{\boldsymbol{N}}{\boldsymbol{N_c}} \tag{9}$$

Where N, N_c denoted the signal data amount and compressed data amount. The SNR of the original signal can be calculated as follow

$$\boldsymbol{SNR} = 10\log(\textstyle\sum_{i=1}^{p} S_i^2 / \sum_{i=1}^{p} w_i^2) \tag{10}$$

The SNR of the de-noise signal can be calculated as follow

$$\boldsymbol{SNR} = 10\log\left(\frac{\sum_{i=1}^{p} s_i^2}{\sum_{i=1}^{p}(sw'-w)_i^2}\right) \tag{11}$$

Where wn' denoted the de-noised signal. The uncontaminated vibration signal shown in fig.10, and the original signal shown in Fig.11, the signal after de-noise shown in Fig.12. The compression ratio of the signal is 1.142, And SNR of the original signal is 13.0733dB, while the SNR of the de-noised signal is 20.3990 dB. We can compare the SNR of original signal and de-noised signal and the SNR of de-noise signal is improved obviously. There for the proposed method should be effective for vibration signal de-noising.

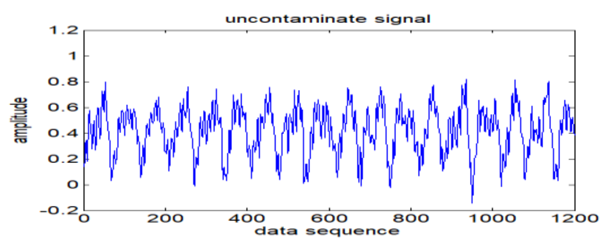

Fig.10. Uncontaminated vibration signal

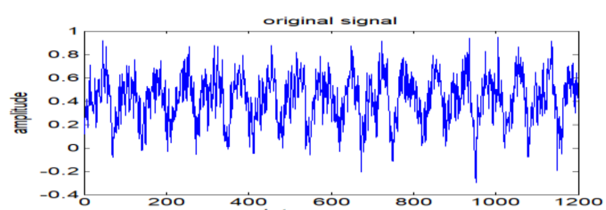

Fig.11. Original signal

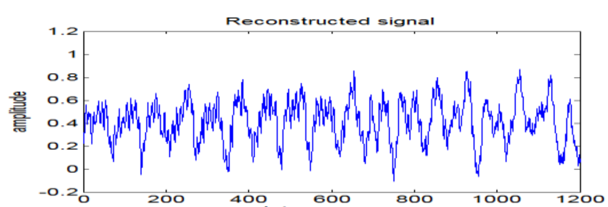

Fig.12. De-noised or reconstructed signal

It can be seen in above figures that de-noised signal by this method is almost closed to the uncontaminated signal $\boldsymbol{S}$ as compared to the original signal. The signal reconstruction error also calculated to evaluate the results as follows [16].

$$\xi = \frac{\|\hat{S} - S\|_2}{\|S\|_2} \tag{12}$$

Where $\widehat{S}, S$ separately denoted the reconstructed signal and original signal. The reconstruct error is 0.0640 when the value of $K = 30$ and $M = 700$ set in the OMP algorithm. The reconstruction error is shown in table 1.

Table 1: Reconstruction error for different value of sparsity K and measurement $M = 600$.

$\boldsymbol{K}$	5	10	15	20	25	30	35	40	45	50
$\boldsymbol{\xi}$	.0448	.0544	.0566	.0603	.0674	.0640	.0601	.0610	.0636	.0690

It is seen in table that when the value of K is increase the reconstruction error will also increase, there for we have to choose the appropriate value of K and M that the results should be better, while when measurement M is increase the error is decrease but execution time will be increase. Therefore setting the measurement $M = 700$ and sparsity $K = 30$ and the results shows above.

5. CONCLUSIONS

This paper proposes a de-noising method for vibration signal based on compressed sensing theory. The experiments results show the performance of the proposed method in vibration signal de-noising. The results of signal de-noising mostly affected by measurement number M and the sparsity K which is set in OMP algorithm. Usually, the value of K smaller, then the de-noising results is better. While increasing the measurements, the de-noising results will improve but corresponding computation time would also be increased. Therefore, we should set moderate M and an appropriate K that the SNR would be improved and signal reconstruction error is minimum. In our experiments, the DCT matrix was used as the dictionary matrix for representing the vibration signal sparsely and Gaussian random matrix used as measurement matrix. The future work mainly focuses on some other transform domain to represent more sparsely which would be beneficial to decrease the signal reconstruction error and also studied recovery algorithm in next work.

REFERENCES

[1]Candes E, Romberg J, Tao T. Robust uncertainty principles: "Exact signal reconstruction from highly incomplete frequency information", IEEE Transactions on Information Theory. 2006, 52(2): 489-509.

[2] Donoho D L. "Compressed sensing", IEEE Transactions on Information Theory, 2006, 52(4): 1289-1306.

[3] Zhang Xinpeng, Hu Niaoqing, Chengzhe, Zhong Hua. "A De-noising Method for Vibration Signal Based on compressed sensing",Prognostic and system Health Management Conference (PHIM-2014 Human), 2014.

[4] Candes E, Wakin M. "An introduction to compressive sensing" [J], IEEE Signal Processing Magazine, 2008, 25(2): 21-30.

[5] Zou Jiancheng, Fan Li. "A Method of Emage De-noising Based on Compressive Sensing" [J], J. NORTH CHINA UNIV. OF TECH, 2012, 24(1): 1-7.

[6] Bhuvneshwar Bajpeyee, S.N. Sharma, "Detection of bearing faults in induction motors using Short Time Approximate Discrete Zolotarev Transform", International Conference on signal processing(ICSP), Nov. 2016.

[7] ZHOU Xiaoxing, WANG Anna SUN Hongying, YANG Hongwu."Speech enhancement based on compressed sensing" [J], J Tsinghua univ(Sci&Tech),2011,51(9): 1234-1238.

[8] Donoho D L, Tsaig Y. "Extention of compessed sensing" [J], Signal Processing, 2006, 86(3): 533-548.

[9]Candes E, Tao T. "Near optimal signal recovery from random projections" Universal encoding strategies [J], IEEE Transactions on Information Theory, 2006, 52(12): 5406-5425.

[10] Piero Espino. J. Rendon,J. Valdez, Y. Davizon, J. Nunez. C. Hinojosa. "Design of a Labview Application for the Teaching of Mechanical Vibration in a Mechatronics Engineering Program",IEEE Latin America Transactions, volume-13, no. 7. (2015).

[11] Baraniuk R. "A Lecture on compressive sensing" [J], IEEE Signal Processing Magazine, 2007, 24(4): 118-121.

[12] J A Trop, A C Gilbert. "Signal recovery from random mea-surements via Orthogonal Matching Pursuit" [J], IEEE Transaction on Information Theory, 2007, 53(12): 4655-4666.

[13] Candes E, Tao T. "Decoding by linear programming", IEEE Transactions on Information Theory, 2005, 51(12): 4203-4215.

[14] Davenport M. "Random Observations on Random on Random Observations: Sparse signal acquisition and processing", PHD thesis, Houston: Rice University, Aug. 2010.

[15] Mallat S. G. and Zhang Z. "Matching pursuits with time-frequency dictionaries" [J], IEEE Trans. on Signal Processing, 1993, 41(12): 3397-3415.

[16] Sai Ji, Liping Huang, Jin Wang, Jian Shen and Jeong-Uk Kim , " An Improved Reconstruction method of Compressive Sensing Data Recovery in Wireless Sensors Networks", International Journal of Security and Its Applications, Vol. 8, No. 1 (2014), pp. 1-8