On Longitudinal Vibrations of a Round Rod Considering Rotatory Inertia

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Abstract. The article considers the solution to the problem of longitudinal oscillations of a round elastic rigidly fixed rod under kinematic excitation of the free end. The results of solving the problem obtained with and without considering rotatory inertia of the body are considered. The application of the finite difference method for solving problems of this kind is substantiated. In this article algorithms have been developed that allow to unambiguously determine the stress-strain state at any point of an arbitrary section of a circular cylindrical rods based on the results of solving the corresponding problems of its vibrations.
Keywords: rod, numerical methods, kinematic excitation, stresses.

1. Introduction

The simplest elements or components of structures usually consist of shells, plates and rods. Rods with a circular cross section are the main elements of many designs. Longitudinal vibrations of elastic rods were studied on the basis of analytical solutions [1,2]. Let us consider a quantitative study of longitudinal vibrations of rods, and the longitudinal waves propagation along the rod arising from a kinematic impact applied to the end of an elastic rod of radius *r*.

The issue of numerical study of longitudinal vibrations of rods has recently acquired great scientific and practical importance [3].

The number of theoretical models for describing longitudinal vibrations of a rod by wave equation with taking into account both transverse displacement and shear deformation described in [4]. Application of numerical methods for calculation different applied problem conformity on the vase of theoretical models for longitudinal rod vibrations with ring defects or drill columns wide used [5-10] also. However, approach for solving of longitudinal vibration with kinematic excitation described below has interest because rotatory inertia was taken at calculation.

2. Statement of the problem

Let us assume that the surface of a rod of length l is free from loads, one end is free, and the other is fixed to a gasket. Let the free end *(z=0)* of the rod be subjected to a longitudinal kinematic load $U(z,t)=\varphi(t)=\mu A\sin\frac{\pi t}{t_1}$ along the rod, where t_1 is the time of the load; *A* is the amplitude.

When a rod vibrates under kinematic excitation, the tip should determine the tension-compression state of the rod that occurs when the load is applied.

To solve the problem, we use the classical equation of longitudinal vibrations of an elastic rod of circular cross section in cylindrical coordinates [2]

$$\rho\mu^{-1}\left(\frac{\partial^2 U_z}{\partial t^2}\right)-E\frac{\partial^2 U_z}{\partial z^2}=0, \qquad (1)$$

where $\mu=\frac{E}{2(1+v)}$; v are Lame's ratio and Poisson's ratio, respectively.

Initial conditions are:

$$U(z,t)|_{t=0}=0 \text{ and } \frac{\partial U(z,t)}{\partial t}|_{t=0}=0 \qquad (2)$$

Boundary conditions are:

$$U(z,t)|_{z=0}=\varphi(t), \quad U(z,t)|_{z=l}=0. \qquad (3)$$

Let us take the following parameters when solving the problem: *t=60s, ν=0.3, ρ =7850, E=2·10*^{11} *, l =50 cm.*

We use the finite difference method to solve the problem. To do this, we divide the rod along the height into parts with a step h, setting the time step τ:

$$D_k=\{z_k=k\cdot h,\quad k=0,\pm1,\pm2,...\pm N,\quad h>0,\quad t_i=i\cdot\tau,\quad i=0,M,\quad \tau>0\}$$

$$h=\frac{l}{N},$$

$$z_k=z_0+k\cdot h, \qquad t_i=t_0+i\cdot\tau, \qquad \tau=\bar{k}\frac{h}{a}, 0<\bar{k}<1. \qquad \bar{k}=0.09,$$

We describe the algorithm for calculating the longitudinal vibration of an elastic rod (1) by the finite difference method. The recursive formula is determined in the same way as above:

$$U_k^{i+1}=\alpha^2U_{k+1}^i+(1-\alpha^2)\left[2U_k^i-U_{k-1}^i\right]_k \qquad (4)$$

where $\alpha^2 = \dfrac{c^2\tau^2}{h^2}$.

The initial conditions are written in the form:

$$U(z,t)\big|_{t=0} = 0,\ U_k^0 = 0$$

$$\frac{\partial U(z,0)}{\partial t} = 0 \Rightarrow \frac{U_k^i - U_k^0}{\tau} = 0 \Rightarrow U_k^1 = U_k^0 = 0$$

$$(k = 0, \pm 1, \pm 2, \ldots \pm N); \tag{5}$$

Boundary conditions are:

$$\begin{aligned} &U(z,t)\big|_{z=0} = \varphi(t), && U_0^i = \varphi^i\ (i = 0, M), \\ &U(z,t)\big|_{z=l} = 0, && U_i^i = 0\ \ (i = 0, M). \end{aligned} \tag{6}$$

We obtain a solution to this problem, considering rotatory inertia. To do this, we use the following equation as the basic oscillation equation.

$$\frac{\partial^2 U}{\partial t^2} - \frac{\partial^2 U}{\partial z^2} + \frac{a_3}{8}\frac{\partial^4 U}{\partial t^2 \partial z^2} = 0, \tag{7}$$

where $a_3 = \dfrac{7q_1^2 - q_1 + 4}{2q_1(1+\nu)}$; $q_1 = 1 - \dfrac{\lambda + 2\mu}{\mu} = -\dfrac{\lambda + \mu}{\mu}$;

After some simplifications of this equation using the finite difference method, we obtain a recursive formula [3]. Here, the third part takes into account the rotatory inertia under the vibration of the rod cross-sections under the following conditions:

$$U_{i+1}^{n+1} + m_k U_i^{i+1} + n_k U_{i-1}^{n+1} = f_k$$

here

$$m_k = \frac{1}{a_k}\left[\frac{1}{\tau^2} + \frac{2\gamma}{\Delta^2} - \frac{a_3(1-2\gamma)}{4\Delta^2\tau^2}\right], \qquad a_k = \left[-\frac{\gamma}{\Delta^2} + \frac{a_3(1-2\gamma)}{8\Delta^2\tau^2}\right],\ n_k = 1,$$

$$f_k = \frac{1}{a_k}\left[\frac{1}{\tau^2}(2U_i^n - U_i^{n-1}) + \frac{\eta}{\Delta^2}(U_{i+1}^{n-1} - 2U_i^{n-1} + U_{i-1}^{n-1}) - \right. \tag{8}$$

$$\left. - \frac{a_3(1-2\gamma)}{8\Delta^2\tau^2}(U_{i+1}^{n-1} - 2U_i^{n-1} + U_{i-1}^{n-1})\right].$$

It is evident from (7) that despite the fact that the equation has the fourth-order, its order in terms of coordinates and time does not exceed the second-order. Therefore, in this case, the initial and final conditions of the problem consist of conditions (5) and (6).

3. Numerical results

Based on the calculations, the results obtained for the classical equation (1) U_z - displacements and σ_{zz} - stresses are presented in the form of graphs in Figs. 1–2. The results obtained on the basis of the refined equation (7) are shown in Figs. 3–5 as time- and coordinate-dependent graphs of displacements U_z.

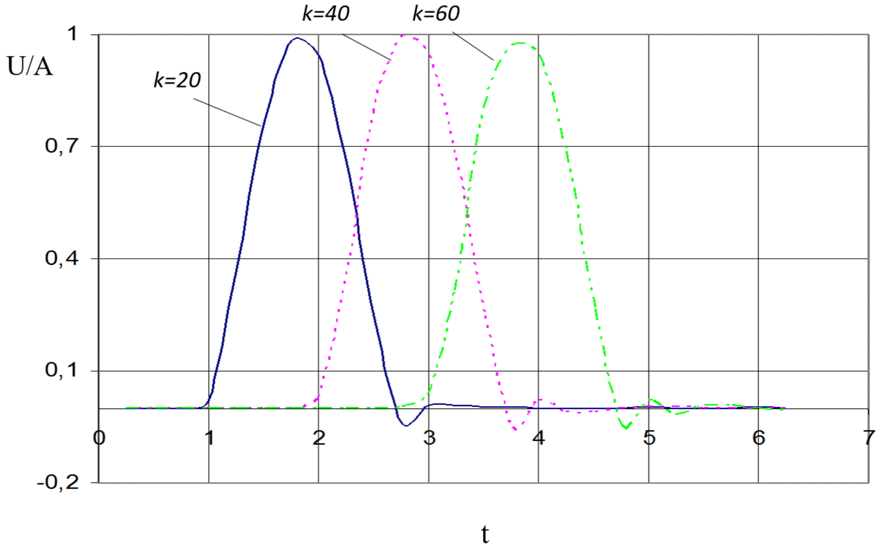

Fig. 1. Change in displacements with time in different sections of the rod

Figure 1 shows:

- the maximum value of the amplitude of longitudinal displacements corresponding to points k=20, 40, 60 in the cross-section of the round elastic rod kinematically excited from the free end does not exceed one (the amplitude of the given kinematic excitation);
- displacements in different sites reach their maximum values at different times, depending on the conditions of the sites.

For example, if the longitudinal displacement of a point in section *k* = 20 reaches its maximum value at time *t* = 1.8, the longitudinal displacement of a point in section *k* = 60 reaches its maximum value at time *t* = 3.8. Therefore, the farther the considered section is from the free end of the rod, the later the longitudinal displacement of the considered point of the section of the tip reaches its maximum value.

The displacement of the points of sections of a round elastic rod is characterized by a very rapid attenuation.

For example, in section *k* = 20, displacement oscillations occur only in the time interval after which there are practically no oscillations in this section.

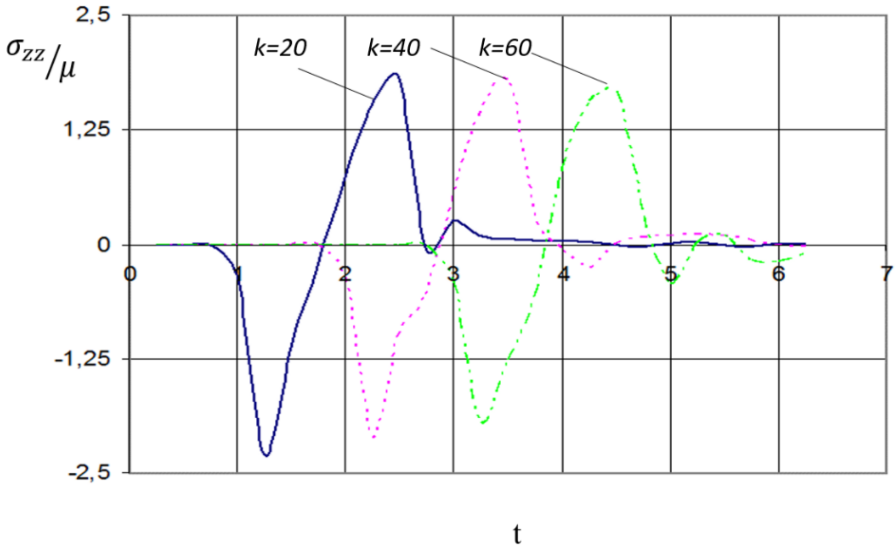

Fig.2. Change in stresses with time in various sections of the rod.

Figure 2 shows that:

- the change in stresses with time in different sections of the rod, kinematically excited from the free end, occurs according to sinusoidal law;
- normal stresses in the rod sections, kinematically suspended from the free end, corresponding to grid points k = 20,40,60, are calculated with time. For example, the maximum stress amplitude in section k = 20 is approximately 1.8, and the amplitude in section k = 60 is approximately 1.65. Therefore, the normal stress is reduced by approximately 19% from k = 20 to k = 60;
- stresses in different sections of the rod have sufficient amplitude only at the initial time when the longitudinal wave reaches these sections, and this condition occurs during a time equal to one oscillation period and fades very quickly with a subsequent increase in time.

4. Discussion of results

Consider the graphs of displacements under kinematic excitation at the free end in various sections of the rod.

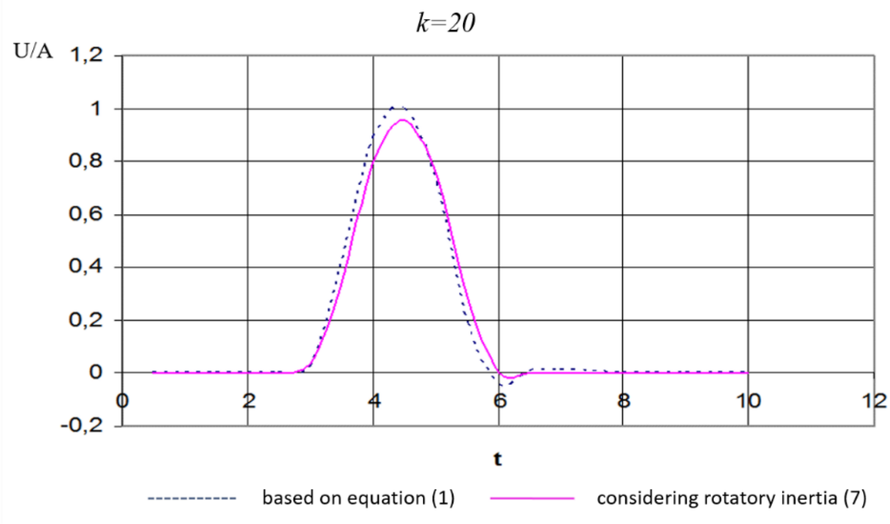

Fig.3. Change of displacement in time in the section of the rod with kinematic excitation, corresponding to k = 20

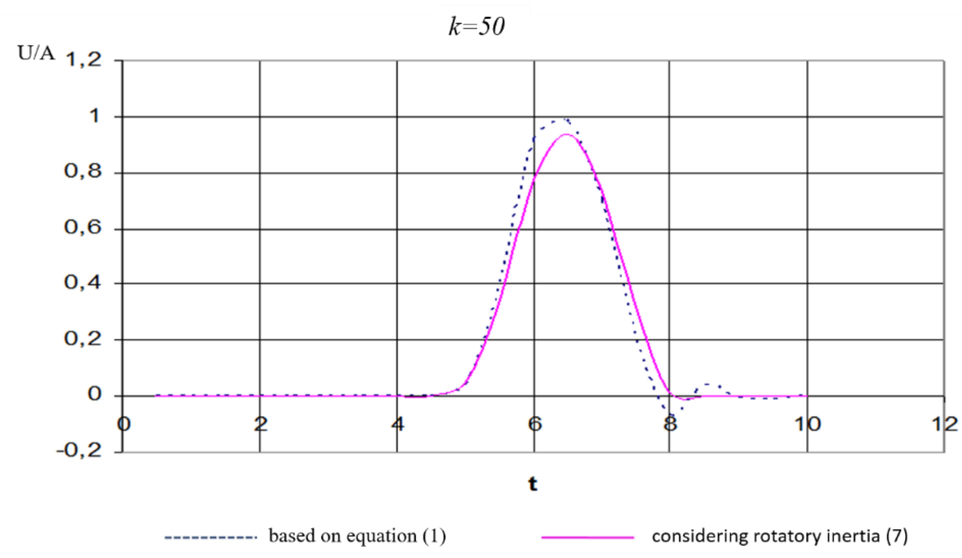

Fig.4. Change of displacement in time in the section of the rod with kinematic excitation, corresponding to k = 50

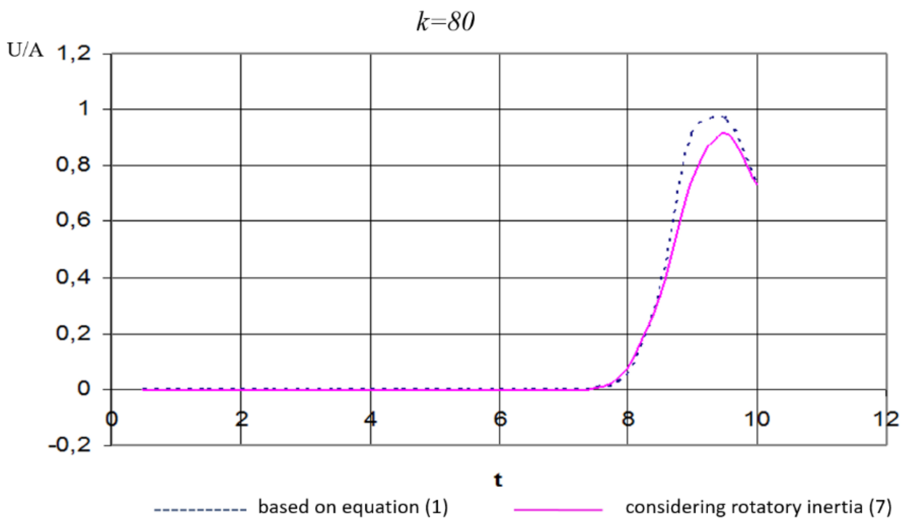

Fig.5. Change of displacement in time in the section of the rod with kinematic excitation, corresponding to *k* = 80

From the graphs (Figs. 3-5), we can draw the following conclusions:

- the influence of rotatory inertia on the displacement amplitude increases with the distance from the end of a round elastic rod;
- the change in stresses in various sections of a round elastic rod, kinematically excited from the free end, calculated on the basis of equation (7), also occurs according to a sinusoidal law, as in the case of equation (1).

5. Conclusions

Longitudinal vibrations of a round rigidly fixed elastic rod with kinematic excitation at the free end were numerically studied on the basis of equations (1) and (7), and the corresponding graphs were plotted for various values of *k*. The normal stresses in the sections of the round elastic rod corresponding to points *k*=20, 40, 60 of the grid attenuated over time. For example, at the highest stress amplitude in section *k*=20, equal to approximately 1.8, the value of the amplitude in section *k*=60 was approximately 1.65. Therefore, the normal stress decreases by approximately 19% when changing the cross section from k=20 to k=60.

From this, it follows that:

1) the influence of rotatory inertia causes gradual damping of excitations for an elastic body both in coordinates and in time;

2) the influence of rotatory inertia in cross sections on displacements and stresses leads to a substantial (to 20%) decrease in the amplitudes of their oscillations.

References

1 Khudoynazarov, K., Algashev, B.F.Y., Mavlonov, T.: Mathematical Modelling of Torsional Vibrations of the Three-layer Cylindrical Viscoelastic Shell. IOP Conf. Ser. Mater. Sci. Eng. 1030, 012098 (2021). https://doi.org/10.1088/1757-899X/1030/1/012098

2 Filippov, I.G., Kudainazarov, K.: General Transverse Vibrations Equations for a Circular Cylindrical Viscoelastic Shell. Sov. Appl. Mech. 26, 351–357 (1990). https://doi.org/10.1007/BF00887127

3 Khudoynazarov Kh Kh, Abdirashidov A, Burkutboyev Sh M 2016 Torsional Vibrations of the Viscoelastic Round Bar Rotating with the Constant Angular Velocity. *Mat. Mod. Chisl. Met.*

4 De Santis, D., Kottapalli, S., Shams, A.: Numerical Simulations of Rod Assembly Vibration Induced by Turbulent Axial Flows. Nucl. Eng. Des. (2018). https://doi.org/10.1016/j.nucengdes.2018.04.027

5 Abirov, R.A.: On the Physical Reliability and Taking Complex Loading Into Account in Plasticity. Mater. Sci. (2008). https://doi.org/10.1007/s11003-009-9114-6

6 Abirov, R.A., Khusanov, B.E., Sagdullaeva, D.A.: Numerical Modeling of the Problem of Indentation of Elastic and Elastic-Plastic Massive Bodies. In: IOP Conference Series: Materials Science and Engineering (2020)

7 Popov, A.L., Sadovsky, S.A.: On the Conformity of Theoretical Models of Longitudinal Rod Vibrations with Ring Defects Experimental Data. Vestn. Samar. Gos. Tekhnicheskogo Univ. Seriya Fiz. Nauk. (2021). https://doi.org/10.14498/vsgtu1827

8 Liu, S., Fu, M., Jia, H., Li, W., Luo, Y.: Numerical Simulation and Analysis of Drill Rods Vibration During Roof Bolt Hole Drilling in Underground Mines. Int. J. Min. Sci. Technol. (2018). https://doi.org/10.1016/j.ijmst.2018.05.018

9 Akhondizadeh, M.: Analytical Solution of the Longitudinal Wave Propagation Due to the Single Impact. J. Low Freq. Noise Vib. Act. Control. (2018). https://doi.org/10.1177/1461348418793122

10 Baragunova, L., Shogenova, M., Kanukoyeva, L.: Longitudinal Vibrations of Rods. E3S Web Conf. (2021). https://doi.org/10.1051/e3sconf/202128101044