EPiC Series in Computing
Volume 103, 2024, Pages 145–152
Proceedings of the 11th Int. Workshop on Applied Verification for Continuous and Hybrid Systems

Stability Verification of an Industrial Switched PI Control Systems

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Abstract

Benchmark Proposal: In this paper, we propose a benchmark that addresses the challenge of verifying Global Asymptotic Stability for a switched control of a turbofan engine. The control switches among two Proportional-Integral (PI) controllers and is parametrized by the reference values used to stabilize the output. We formulate the mathematical model as an affine switched system with a parametric affine term. The verification problems are, on one hand, to prove symbolically the stability of the system for specific reference values, and, on the other hand, to synthesize a region of parameters for which the stability is guaranteed. We report on previous works partially solving these problems.

1 Introduction

In this paper, we describe a mathematical model for the control design of an aircraft engine. The engine model is represented by a linear state space model of 18 internal state variables, 4 outputs, and 3 inputs. The control switches between two PI (Proportional-Integral) controllers, one for thrust control and another for low-pressure compressor spool speed control, based on the engine state and pilot commands. After reformulating the PI controllers in terms of differential equations, we obtain a hybrid system with 21 state variables and two modes, depending on a set of parameters that represent the reference values of the system. We want to study the dependence of the (symbolic) stability of such system on these parameters.

Although the proposed benchmark is a simplified (linearized) version of the original one, it can be considered typical within its field. It exhibits crucial phenomena relevant to control systems design, which directly impact the efficiency and performance of aerospace technologies. As such, this benchmark could serve as a valuable tool for studying and refining control strategies in a practical context.

We are going to present a parametric problem. Fixing the parameters to a specific value leads to a formulation that can be readily adapted to be studied with existing tools. As for the general problem, current tools do not tackle directly this kind of questions. Nevertheless, an extension to work with this parametric problem could be general enough to be a good utility for many situations, and might be considered to advance the applicability of existing tools. The affine dynamics and the switching between two PI controllers offer a precise framework for model construction. Additionally, global asymptotic stability is a basic property to be verified.

Given the safety-critical aspects of the control of aircraft engines, symbolic guarantees appear to be the correct way to approach the problem.

Since the benchmark with 21 state variables is already quite challenging for symbolic techniques, we created also a number of simplified versions that can be useful to evaluate the scalability of the potential verification solutions.

Structure In Sections 2 and 3 we introduce the mathematical tools and manipulations to describe the model. In Section 4 we describe the derivation of the benchmark. Finally, in Section 5, we explictely state the problems and present some results already available.

2 Background

In this section we briefly recall the notation and theory needed to describe the problem.

We write $\mathbb{N}$ for natural and $\mathbb{R}$ for real numbers. We use boldface letters for vectors and we denote the transpose of a matrix A by A^{T}.

2.1 Linear Dynamical Systems

A *continuous-time linear dynamical system with state space* $\mathbb{R}^n$, denoted by $\mathcal{S} = (A, B, C)$, is defined as the following pair of equations

$$\begin{cases} \dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} \\ \boldsymbol{y} = C\boldsymbol{x} \end{cases} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the *state vector*, $\dot{\boldsymbol{x}}$ denotes the time derivative of $\boldsymbol{x}$, $\boldsymbol{u} \in \mathbb{R}^m$ (for some $m \geq 0$) is the *input vector*, $\boldsymbol{y} \in \mathbb{R}^p$ (for some $p \geq 0$) is the *output vector* and A, B and C are matrices of compatible sizes. The first equation in the system (1) describes the evolution of the state vector $\boldsymbol{x}$, while the second equation describes the value of the output $\boldsymbol{y}$ in a given state $\boldsymbol{x}$. A linear system is *autonomous* when $m = 0$ or, equivalently, the system has no inputs.

An *equilibrium point* for a linear system is a state $\boldsymbol{x}_{\text{eq}} \in \mathbb{R}^n$ such that $\dot{\boldsymbol{x}}_{\text{eq}} = 0$.

2.2 PI Controllers

Let $\mathcal{S} = (A, B, C)$ be a linear system, and suppose that the matrix A is not singular, so that the system has a single equilibrium point.

Given a vector of *reference values* $\boldsymbol{r} \in \mathbb{R}^p$ for the outputs of $\mathcal{S}$, the corresponding *error vector* is then defined by $\boldsymbol{e} \doteq \boldsymbol{r} - \boldsymbol{y}$. A *PI controller* for $\mathcal{S}$ is defined by imposing an input-output relation of the following form:

$$\boldsymbol{u} = K_P\boldsymbol{e} + K_I \int_0^t \boldsymbol{e}(\tau)\,\mathrm{d}\tau \tag{2}$$

where K_P and K_I are matrices realizing appropriate (linear) functions of the instantaneous error $\boldsymbol{e}$.

2.3 Switched Systems

A *switched system* refers to a dynamic system that combines continuous state evolution with discrete "switching" events, which can immediately change the state and/or the system's evolution law. Switched systems are part of the broader class of *hybrid systems* and have been intensively studied in the literature [6].

Suppose that the state space $\mathbb{R}^n$ is partitioned into a finite number of *modes* $(\mathcal{R}_i)_{i\in\mathcal{M}}$. In the following, we assume that the partition is obtained by linear constraints on the state space. We call switching surfaces (or guards) the boundaries that separate the modes.

In each of these regions a differential equation specifies the evolution of the state variable:

$$\begin{cases} \dot{\boldsymbol{x}} = f_i(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{y} = g_i(\boldsymbol{x}) \end{cases} \qquad \text{if } \boldsymbol{x} \in \mathcal{R}_i. \tag{3}$$

Given a starting point $\boldsymbol{x}_s$, a trajectory is a uniformly continuous map that is almost everywhere differentiable $\Phi_{\boldsymbol{x}_s} : [0, T] \to \mathbb{R}^n$, where $[0, T]$ is an interval of $\mathbb{R}$, $\Phi_{\boldsymbol{x}_s}(0) = \boldsymbol{x}_s$, and when $(\Phi_{\boldsymbol{x}_s}(t))'$ is well defined, it coincides with $f_i(\Phi_{\boldsymbol{x}_s}(t), \boldsymbol{u}(t))$ if $\Phi_{\boldsymbol{x}_s}(t) \in \mathcal{R}_i$. Whenever the system trajectory hits a switching surface, the continuous state continues to evolve subject to a different evolution law.

2.4 Stability

We recall some basic definitions of stability.

An equilibrium point $\boldsymbol{x}_{\text{eq}}$ of a dynamical system $\dot{\boldsymbol{x}} = f(\boldsymbol{x})$ is called:

- *stable* if $\forall \varepsilon > 0\ \exists \delta > 0$ such that $\|\boldsymbol{x}(0) - \boldsymbol{x}_{\text{eq}}\| < \delta$ implies $\|\boldsymbol{x}(t) - \boldsymbol{x}_{\text{eq}}\| < \varepsilon$ for every $t \geq 0$;
- *asymptotically stable* if it is stable and δ may be taken such that $\|\boldsymbol{x}(0) - \boldsymbol{x}_{\text{eq}}\| < \delta$ implies that $\boldsymbol{x}(t)$ converges to $\boldsymbol{x}_{\text{eq}}$ for $t \to \infty$;
- *globally asymptotically stable* or *GAS* if it asymptotically stable with $\delta = \infty$;
- *exponentially stable* if there exist positive reals c, K and λ such that $\|\boldsymbol{x}(0) - \boldsymbol{x}_{\text{eq}}\| < c$ implies $\|\boldsymbol{x}(t) - \boldsymbol{x}_{\text{eq}}\| \leq K \|\boldsymbol{x}(0) - \boldsymbol{x}_{\text{eq}}\| e^{-\lambda t}$.

In general, exponential stability implies asymptotic stability; for a linear system the opposite implication also holds, so the two concepts are logically equivalent.

3 Switched Systems with Parametric Affine Term

3.1 Switched PI Controller

The benchmark that we propose in this paper focuses on *switching* controllers, where the linear functions within the feedback law (2) can vary based on *switching conditions* defined as linear inequalities regarding the system's outputs.

Thus, we have a linear system $\mathcal{S} = (A, B, C)$ as in (1) with fixed (A, B, C), but the matrices K_P and K_I appearing in equation (2) are substituted by a pair of finite sets of matrices,

$$(K_{I,i})_{i\in\mathcal{M}} \qquad (K_{P,i})_{i\in\mathcal{M}} \tag{4}$$

where $\mathcal{M}$ is the set of operating modes of the switching controller.

In particular, suppose that t_0 is the moment where a switching between mode i_0 and i_1 occurs. In the new mode $i_1 \in \mathcal{M}$, the input-output relation becomes

$$\boldsymbol{u}(t) = K_{P,i_1}\boldsymbol{e}(t) + K_{I,i_1}\int_{t_0}^{t}\boldsymbol{e}(\tau)\,\mathrm{d}\tau + \boldsymbol{c}. \tag{5}$$

The term $\boldsymbol{c}$ is a vector that is constant when no switch occurs. It is set to zero at $t = 0$ and is reset whenever a switching occurs in order to ensure the continuity of the control.

3.2 Reformulation into a Switched Autonomous System

Let $\mathcal{S} = (A, B, C)$ be the open-loop linear system and $\pi = (K_{P,i}, K_{I,i})_{i\in\mathcal{M}}$ the associated switching PI controller.

The closed-loop system resulting from the feedback connection between S and π can be represented by an autonomous switched system in the following manner.

- The *state space* is $\mathbb{R}^{n+m}$, and the state vector represents the concatenation of the state vector $\boldsymbol{x} \in \mathbb{R}^n$ and the input vector $\boldsymbol{u} \in \mathbb{R}^m$: $\boldsymbol{w} \doteq \begin{pmatrix}\boldsymbol{x}\\ \boldsymbol{u}\end{pmatrix}$;
- The set of modes, denoted as $\mathcal{M}$, coincides with the set specified by the switching controller π. The corresponding partition of the state space is obtained by reinterpreting the conditions on the original state space as conditions on $\mathbb{R}^{n+m}$ without involving the last m coordinates $w_{n+1} = u_1, \ldots, w_{n+m} = u_m$.
- The *flow* within region $\mathcal{R}_i$ ($i \in \mathcal{M}$) is defined by the differential equations of the original linear system (with $\boldsymbol{u}$ reinterpreted as a state variable),

  $$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} \tag{6}$$

  along with the additional equations derived by differentiating both sides of the PI control relation with respect to time (5) (assuming constant reference values):

  $$\dot{\boldsymbol{u}} = -K_{P,i}\dot{\boldsymbol{y}} + K_{I,i}(\boldsymbol{r} - \boldsymbol{y}) \tag{7}$$

  Substituting $\boldsymbol{y} = C\boldsymbol{x}$ and rearranging we get

  $$\dot{\boldsymbol{u}} = -K_{P,i}C\dot{\boldsymbol{x}} - K_{I,i}C\boldsymbol{x} + K_{I,i}\boldsymbol{r} \tag{8}$$

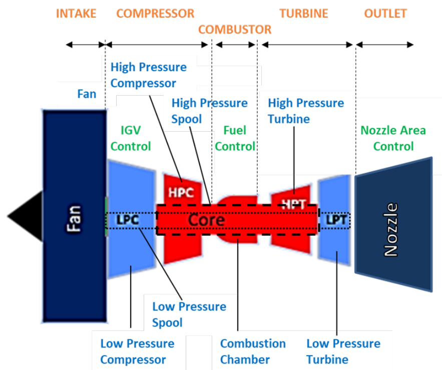
  and using equation (6) we obtain finally

  $$\dot{\boldsymbol{u}} = (-K_{P,i}CA - K_{I,i}C)\boldsymbol{x} - K_{P,i}CB\boldsymbol{u} + K_{I,i}\boldsymbol{r} \tag{9}$$

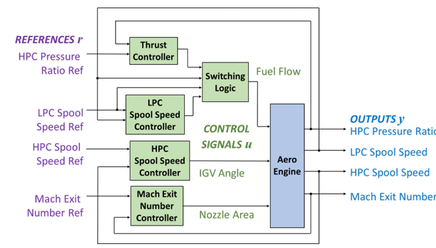
  In terms of the vector $\boldsymbol{w}$, the system of ordinary differential equations consisting of equations (6) and (9) can be written more compactly as

  $$\dot{\boldsymbol{w}} = \begin{pmatrix} A & B \\ N_i & M_i \end{pmatrix}\boldsymbol{w} + \begin{pmatrix} 0 \\ K_{I,i} \end{pmatrix}\boldsymbol{r} \tag{10}$$

  where we have defined $N_i \doteq -K_{P,i}CA - K_{I,i}C$ and $M_i \doteq -K_{P,i}CB$.
- The outputs of the reformulated systems are just the outputs of S, extended to the new state vector in a trivial way (without any reliance on $\boldsymbol{u}$): $\boldsymbol{y} = C\boldsymbol{x} + 0\boldsymbol{u}$.



(a) An example of a dual spool engine.



(b) The engine control model.

Figure 1: Schemas of the model, taken from [2].

4 Case Study

4.1 Basic Engine Operation

A basic outline of an aircraft turbofan engine is depicted in Figure 1a. The main components, in order from left to right, are: the inlet (engine front), the compressor (low and high pressure stages), the combustion chamber, the gas turbine (high and low stages), and the exhaust nozzle (back of the engine). The aircraft engine ensures a steady flow of air into the pressure vessel, undergoing a two-stage compression process (low and high pressure) and combining with fuel within the combustion chamber to generate thrust. The engine control system oversees various control sections to manage fundamental stages of intake, compression, combustion, and exhaust. Throughout these operations, in addition to meeting performance criteria, strict adherence to critical safety parameters is essential. These parameters encompass measures to prevent engine surge and stall, maintain combustion chamber temperatures within safe limits, and more. Therefore, it is imperative to rigorously verify and certify any control approach implemented on the engine's embedded controllers to meet specific safety standards.

4.2 Use Case Description

We here recall the description of a jet engine control system presented in [2]. Specifically, the control design challenge for a turbofan engine discussed in [10, 9] is tackled using single-input single-output PI controllers [4], in contrast to the multivariable controllers proposed in [10, 9]. The decision to adopt the aforementioned control architecture is driven by the simplicity of PI controllers. This choice facilitates a lean control implementation with minimal complexity, thereby establishing a practical scenario that is easily understandable even to individuals without expertise in control theory.

The control system under investigation is depicted in Figure 1b. It contains four PI controllers responsible for regulating the Low Pressure Compressor (LPC) Spool Speed, High Pressure Compressor (HPC) Pressure Ratio, Mach Exit Number, and HPC Spool Speed. These quantities are represented as y_0, y_1, y_2, and y_3, respectively, forming the four measured output signals of the engine. Collectively, they constitute the system (engine) output column vector $\boldsymbol{y} = \begin{pmatrix} y_0 & y_1 & y_2 & y_3 \end{pmatrix}^T$. The desired reference values for these engine outputs, denoted as r_0,

r_1, r_2, and r_3, are command signals originating from a supervisory engine management system, grouped as the system reference (command) vector $\boldsymbol{r} = \begin{pmatrix} r_0 & r_1 & r_2 & r_3 \end{pmatrix}^T$. Each reference signal, along with the corresponding engine output, serves as input to a single PI controller, as illustrated in Figure 1b. The output signals from the PI controllers determine the actuation signals for Fuel flow, Nozzle Area at the engine exhaust, and Inlet Gain Vane (IGV) Angle at the HPC, denoted as u_0, u_1, and u_2, respectively, forming the control signal vector $\boldsymbol{u} = \begin{pmatrix} u_0 & u_1 & u_2 \end{pmatrix}^T$. Notably, the Fuel Flow actuation signal u_0 is determined through a mode-switching mechanism between the Thrust and LPC Spool Speed controllers. Specifically, if y_0 is greater than r_0, the Flow Rate is determined by the LPC Spool Speed controller's output; otherwise, it is determined by the minimum of the HPC Pressure Ratio and LPC Spool Speed control outputs. This switching operation ensures safety protection from engine compressor surge instabilities by limiting the LPC Spool Speed (r_0 command).

Conversely, in scenarios where no constraints are imposed on LPC Spool Speed (i.e., $y_0 \leq r_0$), selecting the minimum signal as described above always leads to fuel savings [9]. To ensure continuity of the control, whenever switching occurs between the Thrust and LPC Spool Speed controllers, the integrator of the activated controller is reset in order to match the value of the deactivated controller. The actuation signal u_1 for Nozzle Area is solely determined by the Mach Exit Number controller, while the actuation signal u_2 for IGV Angle is solely determined by the HPC Spool Speed controller.

We utilized the mathematical model of the engine system presented in [10, 9] to formulate a linear system comprising eighteen continuous-time ordinary differential equations. This model is expressed in matrix form of the form (1), where $\boldsymbol{x} \in \mathbb{R}^{18}$ represents the vector of internal variables, and $\boldsymbol{y}$ and $\boldsymbol{u}$ denote the engine output and control signal vectors, respectively, as described earlier. The engine model parameters A, B, and C are constant real matrices of appropriate dimensions, with their exact values available at [1].

The PI controllers' designs in continuous time [4] are given by the equations (5), with $\mathcal{M} = \{0, 1\}$ as the set of modes. The matrices (4) expressing the integral and proportional controller gains can be found at [1]. A tolerance of 1 is considered when comparing the values of y_0 and r_0. Following the functional operation of the control system and the safety switching mentioned earlier, the switching law is:

$$i = \begin{cases} 0 & \text{if } r_0 - y_0 < 1 \\ 1 & \text{otherwise} \end{cases}$$

The two operating regions $\mathcal{R}_0$ and $\mathcal{R}_1$ are then defined as

$$\mathcal{R}_0 = (\boldsymbol{g}^{(0)})^{\mathrm{T}}(\boldsymbol{y} - \boldsymbol{r}) > -1; \qquad \mathcal{R}_1 = (\boldsymbol{g}^{(1)})^{\mathrm{T}}(\boldsymbol{y} - \boldsymbol{r}) \geq 1$$

where $\boldsymbol{g}^{(0)} = (1, 0, 0, 0)$ and $\boldsymbol{g}^{(1)} = (-1, 0, 0, 0)$, with $\mathcal{R}_0$ corresponding to the nominal operation region. Applying the reformulation described in Section 3.2, we obtain a family of autonomous switched systems of the form

$$\dot{\boldsymbol{w}} = \begin{cases} A_0 \boldsymbol{w} + B_0 \boldsymbol{r} & \text{if } \boldsymbol{g}^T(C\boldsymbol{w} - \boldsymbol{r}) < h \\ A_1 \boldsymbol{w} + B_1 \boldsymbol{r} & \text{otherwise} \end{cases}$$

parametrized by $\boldsymbol{r} \in \mathbb{R}^4$.

Exploiting the fact that, by construction, $\boldsymbol{g}^{\mathrm{T}} C A_0^{-1} B_0 = -\boldsymbol{g}^{\mathrm{T}}$, we can, without changing the stability properties of the system, translate every system by $A_0^{-1} B_0 \boldsymbol{r}$, obtaining the simpler form

$$\dot{\boldsymbol{w}} = \begin{cases} A_0 \boldsymbol{w} & \text{if } (\boldsymbol{g}^T C)\boldsymbol{w} < h \\ A_1 \boldsymbol{w} + L\boldsymbol{r} & \text{otherwise} \end{cases} \tag{11}$$

where $L = B_1 - A_1 A_0^{-1} B_0$. Also in this case the family of systems is parametrized by $\boldsymbol{r} \in \mathbb{R}^4$.

The exact values of $A_0, A_1, L, \boldsymbol{g}^T C, h$ can be found at [1].

5 The problem

The system in (11) is the autonomous version of the closed loop described in Section 4. In the following, we state questions on this reformulation. However, equivalent questions can be stated on the non-autonomous version of the system.

Problem 1. For the system described in (11), fix a value of the parameter vector $\boldsymbol{r} = \bar{\boldsymbol{r}}$. Symbolically verify the stability of the system for this value.

Problem 2. For the system described in (11), describe the region

$$G = \{\boldsymbol{r} \in \mathbb{R}^4 \text{ such that the corresponding system is GAS.}\}.$$

One of the difficulties in the approach of both problems is represented by the high number of variables, 21, in the state space. For this reason, obtaining proofs of correctness must deal with symbolic techniques and will most-likely need optimized methodologies in order to manipulate the data.

To evaluate the scalability of the methods, we provide reduced models alongside the original one, with state variable of dimension 6, 8, 13, and 18 respectively, obtained by using *Balanced Truncation Model Reduction* on the original system. Notice that no relation can be inferred between the region G for the original system and the ones for the reduced systems.

We also provide an integer version of the matrices, obtained by truncating the original system. Also in this case, this version should be considered as a test to evaluate tools on a more specific type of input, that does not imply any result on the original problem.

5.1 Results Obtained

5.1.1 Synthesis of single mode Lyapunov function with fixed parameter

Problem 1 is tackled in [2] for the parameter vector $\bar{\boldsymbol{r}} = [0.5; 5; -1; 20]$. Stability is proven in each mode using Lyapunov methods. In doing so, different numerical methods are compared, thus providing interesting insights on their reliability. In particular, Lyapunov functions are synthesized either solving a matricial equation symbolically, or solving an LMI (Linear Matrix Inequality) problem numerically and validating it symbolically. Several LMI variations are considered to compare their robustness. The performance of different LMI solvers (mosek, cvxopt, smcp) is also taken into account.

The validation is exploited through the use of the symbolic package SymPy, mathematica, or by an SMT solver. Also in this case, several SMT solvers (cvc5, z3) are compared.

Also the synthesis of a region of the state space that is guaranteed to converge for the switched system is considered. Elaborating on the robustness of such a region in relation to a change in the reference value $\boldsymbol{r}$, the authors derive bounds on the variation of parameters for which we are guaranteed to stabilize without switching control.

5.1.2 Under-approximation of GAS parameter region

Problem 2 is considered in [3], synthesizing a region of parameters where Global Asymptotic Stability (GAS) for the switched dynamics can be formally proven. The methods used build on top of the work in [5], [7], and [8], which relies on Piecewise Quadratic Lyapunov Functions (PQLF). The extension taken into account involves providing symbolic representations of the proof obligations associated with PQLF to formally verify the GAS arguments.

After fixing a candidate hypercube to be contained in G, the PQLF is adapted to deal with regions of parameters in two ways: looking for a PQLF that is the same in every vertex of the hypercube and proving that by convexity it is a valid PQLF for every internal point of the hypercube; extending the PQLF to a parametric version so that for every internal point of the hypercube, the evaluation of the parametric PQLF on that point provides a valid PQLF for the corresponding switched system. The authors then implement and apply an algorithm that splits an initial bounded region into hypercubes and selects those for which a PQLF on the vertices (which are concrete parameter evaluations) can be found.

The benchmark described here, among others, is considered to evaluate the methodology. Similarly to the previous case, different LMI solvers, SMT solvers, and other validation methods are compared.

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